

Propagating Modes Along a Thin Wire Located Above a Grounded Dielectric Slab

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Abstract—The possible propagating modes supported by a wire located parallel to a grounded dielectric slab are investigated. While at low frequencies, a “quasi-TEM” behavior is exhibited, it is shown numerically that under certain conditions, a very different “surface-attached” character emerges. These results suggest the possibility of similar behavior occurring in the related, but more difficult to analyze configuration of open microstrip lines. The particular structure we analyze here is of interest mainly because of its potential application in air strip ground radar monitoring systems, which conceivably can consist of a horizontal wire located above a reinforced concrete slab lying above a conducting earth surface.

I. INTRODUCTION

IT HAS recently been found [1], [2] that a thin horizontal wire located parallel to a conducting earth can support a so-called “earth-attached” mode in addition to the well-known “transmission line” mode which becomes TEM in the limit of a perfectly conducting earth. The physical mechanism which gives rise to this new mode seems to be an interaction of the wire with the Zenneck surface wave of the air–earth interface. If, instead of a semi-infinite earth, the wire is located above a grounded dielectric slab, as shown in Fig. 1, it seems possible that similar interaction could occur with the TM surface wave of the slab (whose surface wave character is more pronounced than that of the Zenneck wave), and that a second mode could appear in this case as well. Among other possible applications, such a structure conceptually could be a very practical one to use in the development of air strip or perimeter monitoring systems installed above a reinforced concrete slab of finite thickness lying on the top of a conductive earth. In addition it may also give some insight into the structurally similar microstrip transmission line at much higher frequencies. However, with the use of a thin-wire assumption, there is no need to solve an integral equation for the current in the conductor (as is the case with microstrip) before finding the equation for the propagation constant. It can thus be decided by studying the present configuration whether such a phenomenon as conjectured above could be expected in microstrip, before actually going through the analysis. Since the only previous related work seems to have been an investigation of a wire centered in an ungrounded slab [3], in which case, no TEM mode exists in the low frequency limit, the analysis of the present problem also seems desirable for the better understanding of other problems involving, for example, the

properties of two-wire transmission lines partially filled with dielectric.

II. FORMULATION OF THE MODAL EQUATION

Let the thickness of the slab be t , and its relative permittivity $\epsilon_r = n^2$. The wire, whose radius is a , is located at a height d above the surface of the slab. The characteristic equation for propagating modes can be derived in a thin-wire approximation by following the procedure of Wait [4] who treated a wire over the earth. We shall therefore omit most of the detail in the derivation, touching only high points and presenting the final result.

It is assumed that a current $I \exp(ik_0 \alpha z - i\omega t)$ is flowing in the wire, where $k_0 = \omega(\mu_0 \epsilon_0)^{1/2}$ is the free space propagation constant, and α is the normalized propagation constant of a mode on the wire. The z -axis coincides with the axis of the wire, while the x - and y -coordinates in the transverse plane are indicated in Fig. 1. The field of this current can be given in terms of the Whittaker potentials (z -components of electric and magnetic Hertz vectors) U and V as

$$\begin{aligned} E_x &= \frac{\partial^2 U}{\partial x \partial z} + i\omega \mu_0 \frac{\partial V}{\partial y}, & H_x &= \frac{\partial^2 V}{\partial x \partial z} - i\omega \epsilon \frac{\partial U}{\partial y} \\ E_y &= \frac{\partial^2 U}{\partial y \partial z} - i\omega \mu_0 \frac{\partial V}{\partial x}, & H_y &= \frac{\partial^2 V}{\partial y \partial z} + i\omega \epsilon \frac{\partial U}{\partial x} \\ E_z &= (k^2 + \partial^2/\partial z^2)U, & H_z &= (k^2 + \partial^2/\partial z^2)V. \end{aligned} \quad (1)$$

The potentials U and V can be given as Fourier integrals with respect to y as follows (I is chosen to eliminate a constant appearing outside all the integrals, and the propagation factor $\exp(ik_0 \alpha z - i\omega t)$ is understood to appear in all field quantities)

$$\begin{aligned} U &= \cdot \int_{-\infty}^{\infty} [e^{-k_0 u_1 |x-d|} + R(\lambda) e^{-k_0 u_1 (x+d)}] \\ &\quad \cdot e^{+ik_0 \lambda y} \frac{d\lambda}{u_1}, \quad x > 0 \end{aligned} \quad (2)$$

$$\begin{aligned} &= \int_{-\infty}^{\infty} e^{-k_0 u_1 d} [P(\lambda) e^{+k_0 u_2 x} + S(\lambda) e^{-k_0 u_2 x}] \\ &\quad \cdot e^{+ik_0 \lambda y} \frac{d\lambda}{u_1}, \quad -t < x < 0 \end{aligned} \quad (3)$$

$$V = \int_{-\infty}^{\infty} e^{-k_0 u_1 (x+d)} M(\lambda) e^{+ik_0 \lambda y} \frac{d\lambda}{u_1}, \quad x > 0 \quad (4)$$

$$\begin{aligned} &= \int_{-\infty}^{\infty} e^{-k_0 u_1 d} [N(\lambda) e^{+k_0 u_2 x} + Q(\lambda) e^{-k_0 u_2 x}] \\ &\quad \cdot e^{+ik_0 \lambda y} \frac{d\lambda}{u_1}, \quad -t < x < 0 \end{aligned} \quad (5)$$

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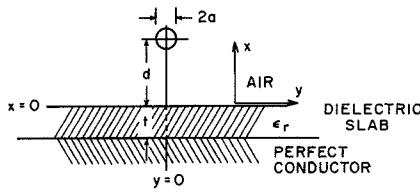


Fig. 1. Geometry of wire over grounded slab.

where, in order to assure convergence of the integrals,

$$u_1 = (\lambda^2 - \zeta^2)^{1/2}, \quad u_2 = (\lambda^2 - \zeta_n^2)^{1/2}, \quad \operatorname{Re} u_1 \geq 0 \quad (6)$$

and $\zeta^2 = 1 - \alpha^2$, $\zeta_n^2 = n^2 - \alpha^2$. The sign of u_2 will be irrelevant since all functions encountered here are even in u_2 .

By enforcing continuity of tangential \bar{E} and \bar{H} at $x = 0$, and the vanishing of tangential \bar{E} at $x = -t$, a system of equations for the unknowns P, R, S, M, N , and Q is obtained. In particular, after some algebra, it is found that

$$R(\lambda) = -1 + \frac{2u_1 \sinh u_2 T}{\zeta^2} \frac{\zeta_n^2 u_1 \cosh u_2 T + \zeta^2 u_2 \sinh u_2 T}{[u_1 \sinh u_2 T + u_2 \cosh u_2 T][n^2 u_1 \cosh u_2 T + u_2 \sinh u_2 T]} \quad (7)$$

where $T = k_0 t$ is a normalized slab thickness. This could be considered as one of the special cases of a stratified half-space discussed by Wait [5]. The modal equation determining α is found in the thin-wire approximation ($k_0 a \ll 1$, $a \ll d$) by enforcing the boundary condition $E_z = 0$ at a single point on the wire [4] or by setting its average value at the wire surface equal to zero [2]. From (1), (2), and (7), we find

$$\zeta^2 [H_0^{(1)}(\zeta A) - H_0^{(1)}(2\zeta D)] + F(\alpha) = 0 \quad (8)$$

where $H_0^{(1)}$ is the Hankel function of first kind

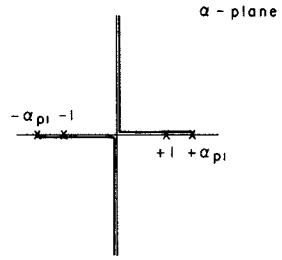
$$A = k_0 a, \quad D = k_0 d$$

and

$$F(\alpha) = \frac{2}{i\pi} \int_{-\infty}^{\infty} \frac{[\zeta_n^2 u_1 \cosh u_2 T + \zeta^2 u_2 \sinh u_2 T] \sinh u_2 T}{[u_1 \sinh u_2 T + u_2 \cosh u_2 T][n^2 u_1 \cosh u_2 T + u_2 \sinh u_2 T]} e^{-2u_1 L} D\lambda \quad (9)$$

The value of ζ is to be chosen so that $\operatorname{Im} \zeta \geq 0$ ensuring a mode which is bounded at infinity in any transverse direction in the air.

As $n \rightarrow \infty$ or $T \rightarrow 0$, it can be seen that $F(\alpha) \rightarrow 0$, and thus the first two terms of (8) represent the effect of a wire and a perfect image at a distance d below the surface of the slab. Similarly, as $n^2 \rightarrow 1$, the terms in (8) can be identified as the wire term and a perfect image term at a distance $d + t$ below the grounding conductor. In any of these limits, the solution is $\zeta^2 = 0$ or $\alpha = \pm 1$, and the mode is a TEM mode traveling on the wire-image transmission line. Clearly $F(\alpha)$ represents the effect of the slab; in fact, the factor $[u_1 \sinh u_2 T + u_2 \cosh u_2 T]$ in the denominator of (9) is, when set equal to zero, the eigenvalue equation for TE surface waves on the grounded slab [6] in the absence of the wire, where u_1 is the attenuation constant in the vertical direction. Likewise, the factor $[n^2 u_1 \cosh u_2 T + u_2 \sinh u_2 T]$ corresponds to TM surface waves.

Fig. 2. Branch cuts in the α -plane.

The integrand of (9) therefore has poles $\pm \lambda_{p1}, \pm \lambda_{p2}, \dots, \pm \lambda_{pN}$, the total number $2N$ of which that lie in the Riemann sheet (6) depends upon the thickness T and refractive index n of the slab. Since the path of integration in the λ -plane lies on the real axis, it is convenient to choose the $+\lambda_{pi}$ to be those with positive imaginary part. Each pole is a function of α , and if α varies such that some pole λ_{pi} crosses the integration path, a discontinuity occurs in $F(\alpha)$ as well as in the

expressions for the fields, so that $\operatorname{Im} \lambda_{pi} = 0$ in fact define a set of branch cuts in the α -plane at $\pm \alpha_{pi}$ as shown in Fig. 2, in addition to the already known pair at $\alpha = \pm 1$. Physically, these cuts correspond to fields radiating away from the wire, along the slab into the given surface wave, at various (possibly complex) angles to the wire, with a normalized wave number α in the z -direction and λ_{pi} in the y -direction. The requirement $\operatorname{Im} \lambda_{pi} \geq 0$ assures that none of the surface waves grow as they move away from the wire. We further note that, independent of the thickness of the slab, there is at least one TM pole in the solution to the slab at any given frequency. Thus it is not difficult to understand why the

transverse field of the wire structure in the presence of the slab could be substantially different from one determined from a quasi-static analysis.

III. APPROXIMATE FORM OF THE CHARACTERISTIC EQUATION

Before we attempt to find the solution of the modal equation as given in (8) on a computer by computing $F(\alpha)$ using numerical integration, we shall derive an analytical form of (8) under a thin-slab approximation which preserves the essential features of interest in the problem, but allows the propagation constants to be found without extraneous numerical computation. Such an approach is not only more efficient computationally, but also can give physical insight into the impact of the surface-wave on the guided wave of a wire above a grounded slab.

If the condition $(n^2 - 1)^{1/2} T \ll 1$ is satisfied, there are no

TE modes on the slab, and only one TM mode (thus only one pair of poles which we denote $\pm \lambda_p$). Under the additional constraint $T^2 \ll D^2$, the damping influence of the exponential in (9) allows us to replace the hyperbolic functions by their small argument forms, giving

$$F(\alpha) \simeq \frac{2}{i\pi} \int_{-\infty}^{\infty} \frac{\zeta_n^2 u_1 + \zeta^2 u_2^2 T}{(u_1 + 1/T)(n^2 u_1 + u_2^2 T)} e^{-2u_1 D} d\lambda \quad (10)$$

which further reduces to

$$\begin{aligned} F(\alpha) & \simeq \frac{2}{i\pi} \int_{-\infty}^{\infty} \left[\frac{1}{u_1 + 1/T} - \frac{\alpha^2}{u_1 + n^2/T} + \frac{\alpha^2 \beta^2}{u_1 - \beta} \right] e^{-2u_1 D} d\lambda \\ & \quad (11) \end{aligned}$$

where

$$\beta = (n^2 - 1)T/n^2 \quad (12)$$

is the approximate location of the value of u_1 for the TM surface wave (these approximations are seen to be related to the surface impedance characterization of a conducting plane with a thin dielectric coating [6]).

The first two terms of (11) can be evaluated as follows:

$$\begin{aligned} \frac{2}{i\pi} \int_{-\infty}^{\infty} \frac{e^{-2u_1 D}}{u_1 + 1/T} d\lambda & = \frac{2}{i\pi} \int_{-\infty}^{\infty} \frac{u_1 T}{u_1(1 + u_1 T)} e^{-2u_1 D} d\lambda \\ & \simeq \frac{2}{i\pi} \int_{-\infty}^{\infty} \frac{\sinh u_1 T}{u_1} e^{-u_1(2D+T)} d\lambda \\ & = H_0^{(1)}(2\zeta D) - H_0^{(1)}(2\zeta(D + T)) \quad (13) \end{aligned}$$

and likewise

$$\begin{aligned} \frac{2}{i\pi} \int_{-\infty}^{\infty} \frac{e^{-2u_1 D}}{u_1 + n^2/T} d\lambda \\ \simeq H_0^{(1)}(2\zeta D) - H_0^{(1)}(2\zeta(D + T/n^2)). \quad (14) \end{aligned}$$

Integrals similar to the last term of (11) are encountered in the wire over earth problem [2], [7], [8], where methods for their approximate evaluation are given. We do not repeat the derivation here, but quote the result, valid under the additional constraint of $|\zeta D| \ll 1$

$$\begin{aligned} \frac{2}{i\pi} \int_{-\infty}^{\infty} \frac{e^{-2u_1 D}}{u_1 - \beta} d\lambda & \simeq 2H_0^{(1)}(2\zeta D) \\ + \frac{4}{i\pi} \frac{\beta}{\sqrt{\alpha^2 - \alpha_p^2}} \left[\pi - \arctan \frac{\sqrt{\alpha^2 - \alpha_p^2}}{\beta} \right] & \quad (15) \end{aligned}$$

where

$$\alpha_p = (1 + \beta^2)^{1/2} \quad (16)$$

denotes the location of the propagation constant for the lowest order TM mode of the slab. The square roots $\sqrt{\alpha^2 - \alpha_p^2}$ are to be chosen with positive real part in order to correspond to $\text{Im } \lambda_p > 0$; note that α^2 must be larger than α_p^2 in order for the surface wave to decay away from the wire. Furthermore, the singularity in (15) at $\alpha = \alpha_p$ should be noted. Now since the existence of this term results only from the excitation of the TM surface wave on the slab, this

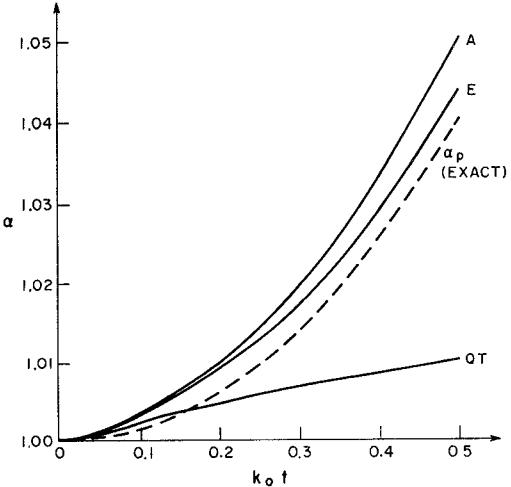


Fig. 3. Comparison of propagation constants α calculated from exact (8)—E; approximate (17)—A; quasi-TEM approximation (18)—Q; $n = 1.5$, $k_0 a = 10^{-4}$, $D = 1.0$. Dotted line indicates exact value of α_p .

singular term simply reflects the dominance of the surface-wave effect whenever the propagation constant α of a guided mode is located near α_p .

By combining (8), (11), (13), (14), and (15), we obtain the approximate modal equation after taking small argument forms for $H_0^{(1)}$:

$$\begin{aligned} 0 = \alpha^2 \ln \frac{2(D + T/n^2)}{A} - \ln \frac{2(D + T)}{A} \\ + 2\alpha^2 \beta^2 \left\{ \ln D \sqrt{\alpha^2 - 1} + \gamma - \frac{\beta}{\sqrt{\alpha^2 - \alpha_p^2}} \right. \\ \left. \left[\pi - \arctan \frac{\sqrt{\alpha^2 - \alpha_p^2}}{\beta} \right] \right\} \quad (17) \end{aligned}$$

where $\gamma = 0.5772 \dots$ is Euler's constant and $\sqrt{\alpha^2 - 1}$ is taken to have positive real part.

Let us seek a perturbation solution of (17). For sufficiently low frequencies, $\beta \rightarrow 0$, and provided that $\alpha^2 - \alpha_p^2$ is not too small, we find the *quasi-TEM approximation*

$$\alpha^2 \simeq \frac{\ln [2(D + T)/A]}{\ln [2(D + T/n^2)/A]} \simeq 1 + \frac{\beta}{D \ln (2D/A)}. \quad (18)$$

Comparing (16) and (18), we see that in order to be a proper mode ($\alpha^2 > \alpha_p^2$) we must have

$$\beta D \ln (2D/A) < 1 \quad (19)$$

which is certainly satisfied in the low frequency limit.

If (19) is not satisfied, the term which is singular at α_p may be important and should not then be neglected. A numerical solution of (17) must then be obtained.

IV. NUMERICAL RESULTS

In order to test the validity of the approximate modal equation (17), its numerical solution was compared with a numerical solution of the exact modal equation (8) using $F(\alpha)$ obtained by a numerical integration of (9). The latter E, as well as the solution of (17), A, is shown in Fig. 3 for $n = 1.5$

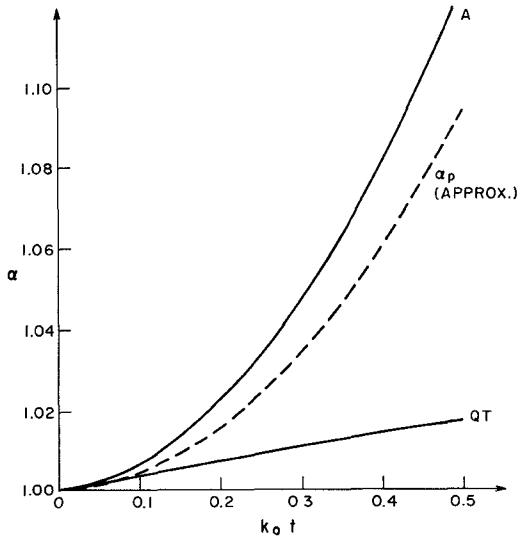


Fig. 4. Propagation constants from (17)—A; and quasi-TEM (18)—QT; $n = 3.0$, $D = 1.0$, $k_0 a = 10^{-4}$. Dotted line indicates approximate value of α_p from (16).

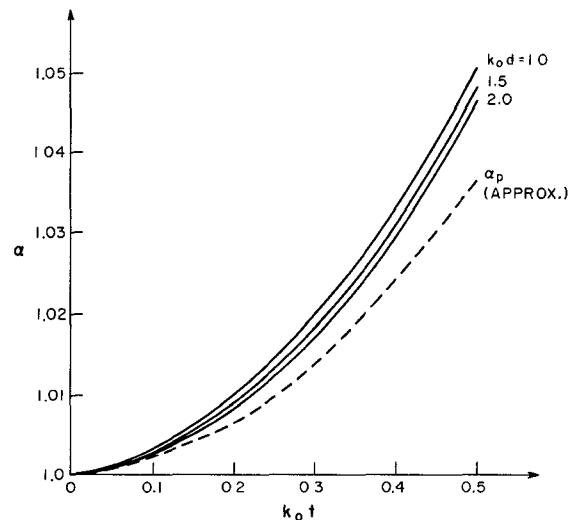


Fig. 5. Propagation constants from (17) for $n = 1.5$, $k_0 a = 10^{-4}$. Dotted line indicates approximate value of α_p from (16).

and $D = 1.0$. Reasonable agreement is found over the entire range of T up to 0.5; indeed, it is rather better than one could expect from the stated approximations. As expected, of course, best agreement is obtained for $T < 0.1$. The quasi-TEM prediction (18) is also displayed QT ; for $T > 0.1$ the error in $(\alpha - 1)$ is on the order of hundreds of percent. The reason for this failure of quasi-TEM theory can be seen by inspecting (17). For small enough T , condition (19) holds and $(\alpha_p - 1)$ is far enough from $(\alpha - 1)$ to leave (18) unaffected. As Fig. 3 shows, however, α as given by (18) soon violates (19), while the actual value of α is “dragged” upwards by the influence of α_p . It is clear that for values of T larger than about 0.2, the mode is heavily influenced by the TM surface wave of the slab, and is no longer given even approximately by quasi-TEM theory. It is thus to be expected that the fields of the mode, when it has attained this “surface-attached” character, will spread out along the slab away from the wire to a much greater extent than those of a quasi-TEM mode. Similar behavior has been found in the wire over earth problem [1], [2].

Fig. 4 shows results for a substrate of higher refractive index ($n = 3$); similar behavior to that of Fig. 3 is observed. In Fig. 5, the (rather small) effect of the value of D on the solution of (17) is shown.

V. LEAKY MODES

It was speculated in the Introduction that because of the surface-wave interaction mechanism present here as well as in the wire over earth situation, a second mode, which appears in the latter case, could also occur here. However, no such second solution of (17) could be found numerically near $\alpha = 1$, α_p , or n . It can be argued that the lossy earth can be obtained from this problem by continuously increasing the loss of n and letting $T \rightarrow \infty$. The question naturally arises: what happens to the second mode? There is, after all,

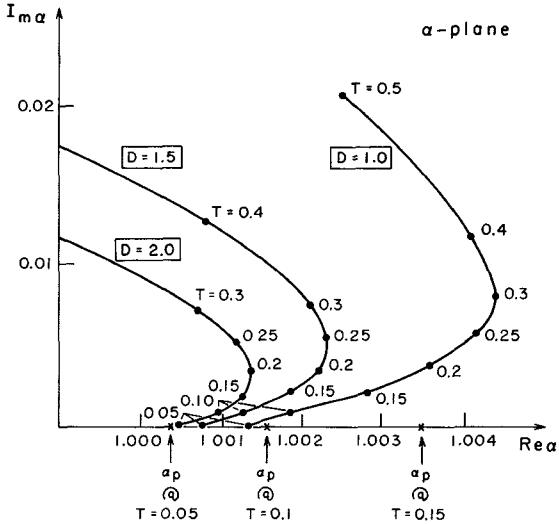


Fig. 6. Propagation constants for leaky modes from (17) for $n = 1.5$, $k_0 a = 10^{-4}$. Modes occur in pairs symmetric about real axis.

a limited number of things that can happen. It seems most likely that the second mode has disappeared into a branch cut, and lies on an improper Riemann sheet, a leaky mode [9]. Unfortunately the continuous transition to the lossy half-space cannot be carried out on (17) because of the thin-slab approximations involved. However, it seems most likely that the cut associated with α_p (which is responsible for the existence of a second mode on the wire over earth) may be “hiding” this leaky mode. A search of the complex α -plane with the sign of $\sqrt{\alpha^2 - \alpha_p^2}$ reversed in (17) (i.e., the Riemann sheet $Im \zeta \geq 0$, $Im \lambda_p \leq 0$) revealed the existence of a leaky mode, with values of α occurring in complex conjugate pairs as is well known for such modes [9]. The trajectories of these modes for $n = 1.5$ are shown in Fig. 6 (only the modes with positive imaginary part are given).

It thus appears that the second mode in the case of a slab

has been forced into the improper Riemann sheet of the α -plane because of the absence of loss. Such a mode, although improper, can be expected to exert some influence in the radiation spectrum of a guiding structure. Although it is impossible to trace this continuous transition under our approximation, the passage of proper modes into improper sheets has been observed in other problems upon continuous variation of the loss [10], [11] or other waveguide parameters [12], [13].

VI. CONCLUSION

The numerical results presented here indicate that strong interaction with the TM surface wave of the slab can occur as the electrical thickness of the slab becomes significant.¹ A second leaky mode has been found which is probably related to the second mode in the wire over earth problem. Both modes are influenced by the singular term in (17) which reflects the surface-wave influence, which thus cannot be neglected in these parameter ranges. A similar singular term has been found in an analysis of microstrip [14] but was neglected in solving the characteristic equation; a recent treatment of optical stripline waveguides [15] notes this singularity as well. The explicit display of this singularity gives physical insight into the reason why neither TEM nor quasi-TEM theory is sufficient for such waveguides operating at significant electrical dimensions. It should be emphasized, of course, that these results are not directly applicable to microstrip (because of the assumption $T^2 \ll D^2$), and that the surface-attached phenomenon is implicit in numerically exact solutions [16].

It should also be noted that in the range where coupling to the surface wave is significant (e.g., T greater than 0.2 in the examples studied here) the proper mode is almost entirely made up of the surface-wave field, which decays as $\exp(-\beta k_0 x)$ above the wire, but only as $\exp(-\sqrt{\alpha^2 - \alpha_p^2} k_0 |y|)$ in the lateral direction. Since, in the case of Fig. 3, for $T = 0.4$, we have $\beta \simeq 0.23$ and $\sqrt{\alpha^2 - \alpha_p^2} \simeq 0.086$ (exact values), it is seen that the mode in a surface-attached condition will be concentrated in the vicinity of the slab, and this property may be useful in guided radar configurations.

¹ The interaction of the wire with the TM surface wave has been interpreted in a more qualitative manner using coupled mode theory [17]. From this viewpoint the uncoupled modes are the quasi-TEM mode of (18) and the TM surface wave on a slab of large but finite width.

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REFERENCES

- [1] R. G. Olsen and D. C. Chang, "New modal representation of electromagnetic waves supported by horizontal wire above dissipative earth," *Electron. Lett.*, vol. 10, pp. 92-94, 1974.
- [2] —, "Electromagnetic characteristics of a horizontal wire above a dissipative earth—Part I: Propagation of transmission-line and fast-wave modes," *Sci. Rep. No. 3* (NOAA Grant no. N22-126-72(G)), Dep. Elec. Eng., Univ. of Colorado, Boulder, 1973.
- [3] V. A. Kaplun and A. A. Pistol'kors, "Diffraction of a plane electromagnetic wave by a cylindrical conductor placed in a plane dielectric layer," *Radiotekh. Elek.*, vol. 9, pp. 1948-1957, 1964 (in Russian; English trans. in *Radio Eng. Electron. Phys.*, vol. 9, pp. 1618-1626, 1964).
- [4] J. R. Wait, "Theory of wave propagation along a thin wire parallel to an interface," *Radio Sci.*, vol. 7, pp. 675-679, 1972.
- [5] —, "Excitation of an ensemble of J parallel cables by an external dipole over an M layered ground," Tech. Rep., Rome Air Development Center, to be published, 1977.
- [6] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, pp. 461-465, 470-474.
- [7] D. C. Chang and J. R. Wait, "Extremely low frequency (ELF) propagation along a horizontal wire located above or buried in the earth," *IEEE Trans. Commun.*, vol. 22, pp. 421-427, 1974.
- [8] D. C. Chang and R. G. Olsen, "Excitation of an infinite antenna above a dissipative earth," *Radio Sci.*, vol. 10, pp. 823-831, 1975.
- [9] T. Tamir and A. A. Ohner, "Guided complex waves Part 1. Fields at an interface," *Proc. Inst. Elec. Eng. (London)*, vol. 110, pp. 310-324, 1963.
- [10] J. W. Cooley and F. Stern, "Solution of the equation for wave propagation in layered slabs with complex dielectric constants," *IBM J. Res. Dev.*, vol. 9, pp. 405-411, 1965.
- [11] Y. N. Lazarenko, V. F. Pavlov, and V. D. Sakhatskiy, "On some properties of surface waves in lossy media," *Radiotekh. Elek.*, vol. 21, pp. 1519-1521, 1976 (in Russian; English trans. in *Radio Eng. Electron. Phys.*, vol. 21, no. 7, pp. 118-120, 1976).
- [12] V. V. Shevchenko, "On the behavior of wave numbers beyond the critical value for waves in dielectric waveguides (media with losses)," *Izv. VUZ Radiofizika*, vol. 15, pp. 257-265, 1972 (in Russian; English trans. in *Radiophys. Quantum Electron.*, vol. 15, pp. 194-200, 1972).
- [13] H. A. Haddad and D. C. Chang, "Field computation of an arbitrarily-oriented dipole above a layered earth," *Sci. Rep. No. 22* (Dep. of Commerce, ITS Contract no. OT-0122), Dep. Elec. Eng., Univ. of Colorado, Boulder, 1977.
- [14] L. J. Van der Pauw, "The radiation and propagation of electromagnetic power by a microstrip transmission line," *Philips Res. Reps.*, vol. 31, pp. 35-70, 1976.
- [15] J. K. Butler, C. S. Wang, and J. C. Campbell, "Modal characteristics of optical stripline waveguides," *J. Appl. Phys.*, vol. 47, pp. 4033-4043, 1976.
- [16] T. Itoh and R. Mittra, "Spectral-domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 496-499, 1973.
- [17] E. Spitz and J. Ernvein-Pecquenard, "Antennes plaquées à rayonnement axial excitées par couplage progressif," *Ann. Radioélectricité*, vol. 20, pp. 25-48, 1965.